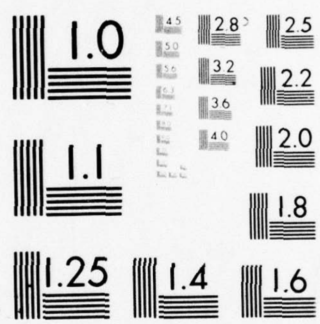
AD-A051 626 CALIFORNIA UNIV SAN DIEGO LA JOLLA DEPT OF APPLIED M--ETC F/G 20/11
A NOTE ON FINITE DEFLECTIONS OF CIRCULAR RING PLATES,(U)
JAN 78 E REISSNER N00014-75-C-0158
UNCLASSIFIED NL

| OF |
AD
A051626

END
DATE
FILMED
4-78
DDC


1.0
4.5
5.0
5.6
6.3
2.8
3.2
3.6
4.0
2.5
2.2
2.0
1.1
1.8
1.25
1.4
1.6

(10)

UNIVERSITY OF CALIFORNIA, SAN DIEGO
Department of Applied Mechanics and Engineering Sciences
La Jolla, California 92093

(6)

A NOTE ON FINITE DEFLECTIONS OF CIRCULAR RING PLATES,

by E. Reissner

(10)

(15) N00014-75-C-0158

(11) Jan[redacted]978

(12) 13p.

Prepared for
OFFICE OF NAVAL RESEARCH
Washington, D.C.

407 513

A NOTE ON FINITE DEFLECTIONS OF CIRCULAR RING PLATES

by

E. Reissner

Department of Applied Mechanics and Engineering Sciences
UNIVERSITY OF CALIFORNIA, SAN DIEGO
La Jolla, California 92093

ABSTRACT

We solve the problem of the symmetrically deforming edge loaded polar-orthotropic circular ring plate for the case of large deformations of a radially rigid plate, and compare the results with the corresponding solutions in accordance with the small finite deformation equations of von Karman.

A NOTE ON FINITE DEFLECTIONS OF CIRCULAR RING PLATES†

By E. Reissner

Introduction. In what follows we consider once more the problem of rotationally symmetric deflections, with reference to the difference between results derived from equations for small finite symmetric deflections (as a special case of the equations of von Karmán) and equations for large symmetric deflections, as given in [1]. Our principal object in reconsidering this question is the formulation and solution of a specific problem which is of such nature that the difference between the results which follow from the two types of equations comes out to be of qualitative (and quantitative) significance in an easily recognized fashion. In addition to this, we will record the solution for two problems of bending and buckling for which the differences turn out to be of somewhat lesser significance.

Equations for Symmetric Bending of Polar-Orthotropic Plates. We recall that the two simultaneous second order differential equations for meridional slope φ and radial stress resultant H follow from a compatibility equation

$$(r\epsilon_\theta)' - \epsilon_r \cos\varphi = \cos\varphi - 1 \quad , \tag{1}$$

in conjunction with the moment equilibrium equation

† A report on work supported by the Office of Naval Research.

$$(rM_r)' - M_\theta \cos\varphi = rH\sin\theta - rV\cos\varphi \quad . \tag{2}$$

In these equations ϵ_θ and ϵ_r are midplane strains and M_r and M_θ are stress couples in the usual sense, with V being an axial stress resultant, and with the prime indicating differentiation with respect to r.

Equations (1) and (2) are complemented by constitutive equations,

$$\epsilon_r = B_r N_r - B_\nu N_\theta \quad , \quad \epsilon_\theta = B_\theta N_\theta - B_\nu N_r \tag{3}$$

$$M_r = D_r \varkappa_r + D_\nu \varkappa_\theta \quad , \quad M_\theta = D_\theta \varkappa_\theta + D_\nu \varkappa_r \tag{4}$$

with N_r and N_θ given in terms of H and V, in the form

$$N_r = H\cos\varphi + V\sin\varphi \quad , \quad N_\theta = (rH)' \quad , \tag{5}$$

where, for simplicity's sake, radial distributed loads are considered absent, and with $\varkappa_r$ and $\varkappa_\theta$ given in terms of φ, in the form

$$\varkappa_r = \varphi' \quad , \quad r\varkappa_\theta = \sin\varphi \quad . \tag{6}$$

We further note the axial displacement formula

$$w' = \sin\varphi \quad , \tag{7}$$

in which, as it is in the equilibrium equation (2), an assumption of small meridional strain is implied. We also note that stress resultants as well as stress couples are defined per unit of <u>un</u>deformed length along curves in the midplane of the plate.

For what follows it will be convenient to introduce the stress function variables

$$\Psi = rH \quad , \quad F = rV \tag{8}$$

and it will be assumed that the plate is uniform, so that the constitutive coefficients B and D are independent of r. It is furthermore assumed that there are also no axial distributed loads, so that the load function F comes out to be independent of r.

With this we have, upon introduction of (3) to (6) into equations (1) and (2), the two simultaneous differential equations

$$B_\theta(r\Psi')' - [B_r r^{-1}\cos^2\varphi + B_\nu(\cos\varphi)']\Psi$$

$$- [B_r r^{-1}\sin\varphi\cos\varphi + B_\nu(\sin\varphi)']F = \cos\varphi - 1 \tag{9}$$

$$D_r(r\varphi')' - D_\theta r^{-1}\sin\varphi\cos\varphi = \Psi\sin\varphi - F\cos\varphi \tag{10}$$

Equations (9) and (10) reduce to the corresponding equations (19) and (21) (with $p_r = 0$) in [1], upon specialization to the case of isotropy, for which $B_\theta = B_r = B$, $B_\nu = \nu B$ and $D_r = D_\theta = D$, $D_\nu = \nu D$.

It has earlier been shown, in [2], that equation (9) may be replaced, effectively, by the abreviated equation

$$B_\theta(r\Psi')' - B_r r^{-1}\Psi = \cos\varphi - 1 \tag{9'}$$

and that a corresponding simplification in (10), which would consist in replacing $D_\theta \sin\varphi\cos\varphi$ by $D_\theta\varphi$, might not be equally appropriate [2]. However, for the problems considered in what follows this simplification will in fact turn out to be admissible as well.

The Boundary Value Problem. We consider a ring plate with inner edge $r = r_i$ and outer edge $r = r_o$ and we assume that the two edges are acted upon by axial stress resultants V_i and V_o and radial stress resultants H_i and H_o. We further assume that no bending moments are applied along either edge. We then have in equations (9) and (10) as expression for F

$$F = r_o V_o = r_i V_i \tag{11}$$

and the boundary conditions for the fourth-order differential equation system (9) and (10) are of the form

$$\Psi(r_i) = \Psi_i \quad , \quad \Psi(r_o) = \Psi_o \tag{12}$$

$$D_r \varphi'(r_i) + D_\nu \frac{\sin\varphi(r_i)}{r_i} = 0 \quad , \quad D_r \varphi'(r_o) + D_\nu \frac{\sin\varphi(r_o)}{r_o} = 0 \quad . \tag{13}$$

We will not, in what follows, attempt to solve the problem in the above generality. Instead, we consider the case of a plate having a limiting-type of orthotropy, of such nature as to allow a relatively simple determination of the differences of the solution of the given problem, in comparison with the results which follow upon replacing equations (9) and (10) by the corresponding von Karmán equations

$$B_\theta (r\Psi')' - B_r r^{-1}\Psi = -\frac{1}{2}\varphi^2 \quad , \tag{14}$$

$$D_r (r\varphi')' - D_\theta r^{-1}\varphi = -F + \Psi\varphi \quad , \tag{15}$$

with the conditions (13) being replaced by conditions of vanishing $D_r\varphi' + D_\nu r^{-1}\varphi = 0$ for $r = r_i, r_o$.

Closed-Form Solutions for Radially Rigid Ring-Plates. The case of a radially rigid plate is given upon setting in the constitutive equations (3) and (4)

$$B_r = B_\nu = 0 \quad , \quad D_r = \infty \quad . \tag{16}$$

With $D_r = \infty$ we have then $\varkappa_r = 0$ and therewith

$$\varphi = \varphi_o \quad , \tag{17}$$

with the moment M_r becoming a reactive quantity. Introduction of (16) and (17) into the differential equations (9) and (10) and observation of the reactive property of M_r in accordance with (2) changes the system of differential equations (9) and (10) into

$$B_\theta(r\Psi')' = \cos\varphi_o - 1 \quad , \tag{18}$$

$$(rM_r)' - D_\theta r^{-1} \sin\varphi_o \cos\varphi_o = \Psi \sin\varphi_o - F\cos\varphi_o \quad , \tag{19}$$

again will the boundary conditions (12) and (13), with the latter conditions now reverting to the form

$$M_r(r_i) = 0 \quad , \quad M_r(r_o) = 0 \quad . \tag{20}$$

In what follows a further simplified version of the above problem will be solved, which is given upon restricting attention to cases for which $r_o - r_i \ll r_i$. We now write

$$r_i = a - b \quad , \quad r_o = a + b \quad , \tag{21}$$

with $3b \ll a$, and we take account of the fact that r in equations (18) and (19) does not differ much from its mean value a by substituting for (18) and (19) the simplified relations

$$aB_\theta \Psi'' = \cos\varphi_o - 1 \quad , \tag{22}$$

$$aM_r' - D_\theta a^{-1} \sin\varphi_o \cos\varphi_o = \Psi \sin\varphi_o - F \cos\varphi_o \quad . \tag{23}$$

Equation (22), in conjunction with the boundary conditions (12), gives as expression for Ψ

$$\Psi = \frac{\cos\varphi_o - 1}{2aB_\theta}[(r - a)^2 - b^2] + \Psi_o \frac{r - (a - b)}{2b} - \Psi_i \frac{r - (a + b)}{2b} \quad . \tag{24}$$

The introduction of (24) into (23), in conjunction with the boundary conditions (20), gives as expression for F in terms of φ_o, Ψ_o and Ψ_i,

$$F = \left(\frac{D_\theta}{a} + \frac{b^2}{aB_\theta} \frac{1 - \cos\varphi_o}{3\cos\varphi_o} + \frac{\Psi_o + \Psi_i}{2\cos\varphi_o}\right) \sin\varphi_o \quad , \tag{25}$$

with M_θ and N_θ now being

$$M_\theta = D_\theta \frac{\sin\varphi_o}{a} \quad , \quad N_\theta = \frac{\cos\varphi_o - 1}{ab}(r - a) + \frac{\Psi_o - \Psi_i}{2b} \quad , \tag{26}$$

and with the corresponding expressions for M_r and N_r leading to stress values which are small compared to the values of the stresses associated with M_θ and N_θ, respectively.

Deflection Due to Equal and Opposite Axial Edge Forces. Setting $\Psi_o = \Psi_i = 0$ and $a + b \approx a$ in equation (25), we obtain as expression for the axial stress resultant

$$V_o = \frac{\sin\varphi_o}{a^2}\left(D_\theta + \frac{b^2}{B_\theta}\,\frac{1-\cos\varphi_o}{3\cos\varphi_o}\right) , \tag{27}$$

with the associated relative deflection w_o of the two edges of the plate being the quantity $2b\sin\varphi_o$.

Expansion of the righthand side of (27) in powers of φ_o, and rentention of the leading nonlinear term gives the approximate result

$$V_o \approx \frac{\varphi_o}{a^2}\left(D_\theta + \frac{b^2}{B_\theta}\,\frac{\varphi_o^2}{6}\right) \tag{28}$$

where now $w_o = 2b\varphi_o$. We note that equation (28) also follows as the exact result of solving the identical problem through use of the small finite-deflection differential equations (14) and (15) and that, with $D_\theta = Eh^3/12$ and $B_\theta = 1/Eh$, this equation may be written in the equivalent form

$$V_o \approx \frac{D_\theta w_o}{2ba^2}\left(1 + \frac{1}{2}\,\frac{w_o^2}{h^2}\right) , \tag{28'}$$

where, as is expected, nonlinearity comes out to be significant for transverse deflections of the order of the plate thickness.

For a comparison of the exact and of the approximate values of V_o, we observe the numerical relations

$$\varphi_o = \frac{\pi}{4}: \quad \sin\varphi_o \frac{1-\cos\varphi_o}{3\cos\varphi_o} \approx 0.098 \quad , \quad \frac{\varphi_o^3}{6} \approx 0.081 \tag{29}$$

which show that the use of small finite-deflection theory for this case amounts to an underestimation of the force necessary to produce an angular deflection of 45° by nearly twenty percent. Equations (27) and (28) show that this percentage error increases steadily with φ_o, in such a way that the exact value of V_o approaches infinity as φ_o approaches $\pi/2$, whereas the approximate value in accordance with (28) remains finite.

Deflection Due to Equal and Opposite Transverse Follower-Forces. The case of transverse edge forces Q_i and Q_o is given upon setting in (25)

$$F = (a + b)Q_o \cos\varphi_o = (a - b)Q_i \cos\varphi_o \quad ,$$

$$\Psi_o = -(a + b)Q_o \sin\varphi_o \quad , \quad \Psi_i = -(a - b)Q_i \sin\varphi_o \quad . \tag{30}$$

Therewith, and again with $a + b \approx a$, equation (25) becomes

$$\frac{Q_o}{\cos\varphi_o} = \frac{\sin\varphi_o}{a^2}\left(D_\theta + \frac{b^2}{B_\theta}\frac{1 - \cos\varphi_o}{3\cos\varphi_o}\right) \quad . \tag{31}$$

Expansion in powers of φ_o and retention of the leading nonlinear term gives the same approximate result (28) for the transverse resultant Q_o as was previously obtained for the axial resultant V_o. However, the difference between Q_o and V_o becomes significant with increasing values of φ_o. In particular, Q_o remains finite as φ_o approaches $\pi/2$, in contrast to the result for V_o.

Buckling Due to Radial Edge Loads. We now set

$$\Psi_o = -(a + b)P_o \quad , \quad \Psi_i = -(a - b)P_i \tag{32}$$

and at the same time set $F = (a+b)V_o = (a-b)V_i = 0$.

Equation (25) now has one trivial solution $\sin\varphi_o = 0$, with $M_\theta = 0$ and $N_\theta = (\Psi_o - \Psi_i)/2b$, and one nontrivial solution, with φ_o related to P_o and P_i in the form

$$(a + b)P_o + (a - b)P_i = \frac{2D_\theta}{a}\cos\varphi_o + \frac{2b^3}{aB_\theta}\,\frac{1 - \cos\varphi_o}{3} . \tag{33}$$

We conclude from equation (33) that buckling will occur for values P_{oc} and P_{ie} given by

$$(a + b)P_{oc} + (a - b)P_{ic} = 2D_\theta a^{-1} \tag{34}$$

Equations (33) and (34) also contain a simple result for the postbuckling behavior of the ring plate (insofar as the assumed deflection pattern persists). We will show this for the case $P_o = P_i$ for which $P_{oc} = D_\theta/a^2$. Equation (33) now becomes

$$P_o = P_{oc}\cos\varphi_o + \frac{2b^3}{a^2B_\theta}\,\frac{1 - \cos\varphi_o}{3} , \tag{35}$$

and this can be solved, so as to give φ_o in terms of P_o/P_{oc} in the post-buckling range, in the form

$$1 - \cos\varphi_o = \frac{(P_o/P_{oc}) - 1}{(2b^3/3B_\theta D_\theta) - 1} \approx \frac{3B_\theta D_\theta}{2b^3}\left(\frac{P_o}{P_{oc}} - 1\right) \tag{36}$$

where, within the range of small-deflection theory, $1 - \cos\varphi_o \approx \varphi_o^2/2$.

REFERENCES

1. E. Reissner, On Finite Deflections of Circular Plates, Proc. Symp. Appl. Math. 1, 213-219, 1949.

2. E. Reissner, On the Equations for Finite Symmetrical Deflections of Thin Shells of Revolution, Progress in Appl. Mech. (Prager Anniv. Volume), 171-178, 1963.

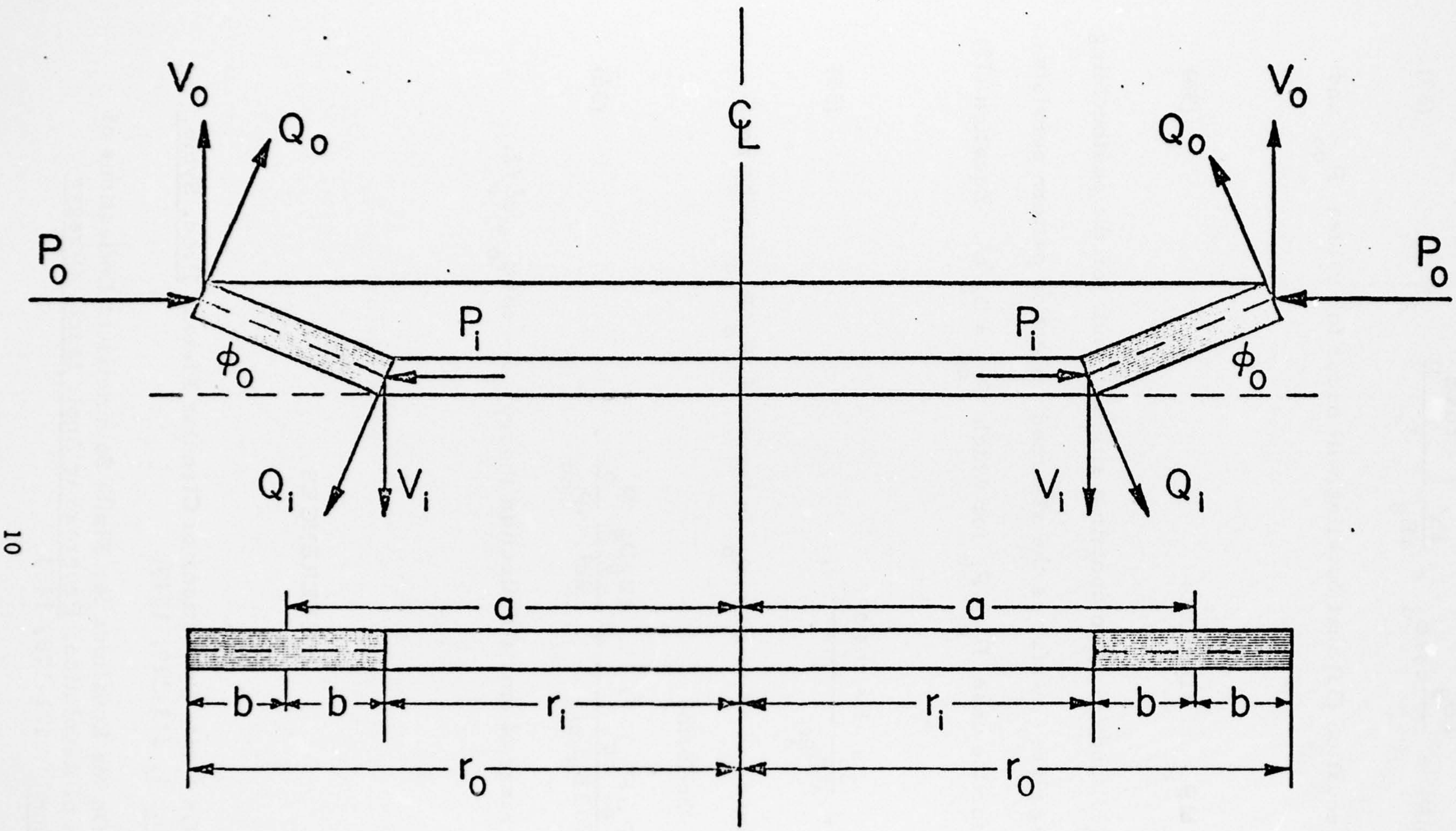


Figure 1. Radially Rigid Circular Ring Plate Acted Upon by (i) Axial, (ii) Transverse, (iii) Radial Edge Load Distributions.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A NOTE ON FINITE DEFLECTIONS OF CIRCULAR RING PLATES.		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) E. Reissner		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0158
9. PERFORMING ORGANIZATION NAME AND ADDRESS Dept. of Applied Mechanics & Engrg. Sciences, University of California, San Diego, La Jolla, California 92093		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Structural Mechanics Branch, Office of Naval Research, Code 474, Arlington, Virginia 22217		12. REPORT DATE January 1978
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Elastic plates, polar-orthotropy, radial rigidity, von Karman equations, large finite deflection equations.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

THIS REPORT ~~We~~ solve the problem of the symmetrically deforming edge loaded polar-orthotropic circular ring plate for the case of large deformations of a radially rigid plate, and compare the results with the corresponding solutions in accordance with the small finite deformation equations of von Karman.

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)